Physical-Layer Multicasting for MISO Downlink:
Beyond Transmit Beamforming

Wing-Kin (Ken) Ma

Department of Electronic Engineering,
The Chinese University of Hong Kong, Hong Kong

A talk presented in MIIS Xi’an, July 2012

Acknowledgement: Xiaoxiao Wu, Anthony Man-Choo So
Outline

• Background
  – physical-layer multicasting problem statement
  – something nobody has reported (to our best knowledge)
  – transmit beamforming and its design via semidefinite relaxation

• Rank-2 beamformed Alamouti space-time block coding

• Stochastic beamforming

• Conclusion
Background
Physical-Layer Multicasting

Nowadays, there is an explosive growth of multimedia services:

- live mobile TV
- multiparty video conferencing
- multimedia streaming for a group of paid users

Physical-layer multicasting: an important class of techniques for resource-efficient massive content delivery
Physical-Layer Multicasting

**Aim:** Send common information to a selected group of users.

- Channel state information at the transmitter (CSIT) is exploited.

- Sounds easier than multiuser unicast (multicast has no interference), but has caveats.
Scenario Setting

- **Scenario:** $M$-user MISO downlink

- **Signal model:**
  \[ y_i(t) = h_i^H x(t) + n_i(t), \quad t = 1, 2, \ldots, T, \]

  where
  
  - $y_i(t)$: rx signal of user $i$ at time $t$;
  - $T$: data frame length;
  - $h_i \in \mathbb{C}^N$: channel from the base station to user $i$;
  - $x(t) \in \mathbb{C}^N$: multi-antenna tx signal carrying the common information;
  - $n_i(t)$: noise following $\mathcal{CN}(0, 1)$.

- **Assumptions:**
  
  - Slow fading: the channels $h_i$ are static over the whole data frame.
  - Perfect CSIT: $h_i$ are known at the base station.
Multicast Capacity

- Let $\Sigma = \mathbb{E}[x(t)x^H(t)]$ be the transmit covariance.

- The multicast capacity is

\[
\text{(MC)} \quad C_{MC}(P) = \max_{\Sigma \in \mathbb{H}^N} \min_{i=1,\ldots,M} \log(1 + h_i^H \Sigma h_i)
\]

\[
\text{s.t. } \Sigma \succeq 0, \quad \text{Tr}(\Sigma) \leq P,
\]

where $P$ is the maximum allowable transmit power.

- (MC) is convex; it can be reformulated as a semidefinite program (SDP).

- The optimal solution of (MC), denoted herein by $\Sigma^*$, may have $\text{rank}(\Sigma^*) > 1$.

- **Question:** How should the physical-layer scheme be designed so that (MC) can be approached in practice?
Some may argue that (MC) can practically be approached by a beamformed (or precoded) spatial multiplexing scheme.

**Beamformed spatial multiplexing:** Let $r = \text{rank}(\Sigma^*)$, and $F \in \mathbb{C}^{N \times r}$ such that $FF^H = \Sigma^*$. Transmit

$$x(t) = Fs(t)$$

where $s(t) \in \mathbb{C}^r$ is an $r$-stream data symbol vector, with $E[s(t)s^H(t)] = I$.

Note: we don’t (and we can’t) assume Gaussian $s(t)$ in realizable physical layer designs.

QAM symbol constellations, coupled with bit interleaved code modulation (BICM), are applied on $s(t)$.

Does this work?
Beamformed Spatial Multiplexing for Multicasting?

Does the beamformed spatial multiplexing scheme

\[ x(t) = Fs(t), \]

where \( FF^H = \Sigma^* \) and \( s(t) \in \mathbb{C}^r \) is an i.i.d. symbol vector, work?

- Arguments from those who support spatial multiplexing:
  - \( E[x(t)x^H(t)] = FF^H = \Sigma^* \); i.e, MC optimal covariance achieving;
  - in a point-to-point MIMO, fast fading scenario, BICM spatial multiplexing has been empirically found to be promising [Hochwald-ten-Brink’03].

- Arguments from those who do not support spatial multiplexing:
  - \( E[x(t)x^H(t)] = \Sigma^* \) is necessary for achieving MC, but not sufficient;
  - finite constellations can lead to results different from what one expects for Gaussian input, except for the single stream case (\( r = 1 \)) probably;
  - the receivers deal with a \( 1 \times r \) underdetermined MIMO detection problem.
How Does BF Spatial Multiplexing Work in Practice?

Bit error rate (BER) performance in a 8-tx-antenna, 32-user, multicast system. The results shown are averages of $r = 4$ instances. BICM, and a rate-1/3 Turbo code, with a code length of 2880 bits, are applied. We also try beamformed quasi-orthogonal space-time block code (QOSTBC), and single-stream beamforming.
Additional Notes on Beamformed Spatial Multiplexing

- The previous simulation result shows that there is a gap between realizable physical-layer designs and information theoretic capacity results.

- It demonstrates that spatial multiplexing does not work; more carefully speaking, beamformed BICM spatial multiplexing does not work for a *naively chosen beamforming matrix* $F$.

- For a similar reason, it is not clear whether ideas like *waterfilling* and *superposition coding* in point-to-point spatial multiplexing can be applied to the multicast scenario.
Transmit Beamforming for Physical-Layer Multicasting

Rationale: Use a realizable physical-layer scheme, namely, transmit beamforming (BF), to perform multicasting.

- The idea of using tx BF in multicasting can be traced back to [Lopez’02]; also [Sun-Liu’04].

- A popularized approach (at least in SP): semidefinite relaxation [Sidiropoulos-Davidson-Luo’06].
Transmit Beamforming for Physical-Layer Multicasting

- A well-known fact: For a SISO channel, the channel capacity may be approached by applying near-ideal scalar channel codes, such as LDPC and Turbo codes.

- Let us fix the transmit strategy as single-stream BF:

$$x(t) = \sqrt{P}w s(t),$$

where $w \in \mathbb{C}^N$ is a BF vector, $P$ is the maximum allowable transmit power, and $s(t) \in \mathbb{C}$ is a data symbol stream with $\mathbb{E}[|s(t)|^2] = 1$.

- At the rx sides, we have $y_i(t) = \sqrt{P}h_i^H w s(t) + n_i(t)$, which is equivalently a SISO channel with SNRs

$$\text{SNR}_i = P|h_i^H w|^2.$$

- Assuming ideal channel coding on $s(t)$, we obtain an achievable multicast rate

$$C_{BF}(w, P) = \min_{i=1,\ldots,M} \log(1 + \text{SNR}_i).$$
Multicast Beamforming Design

- **Problem:** Maximize the beamforming multicast rate; i.e.,

\[
\max_{\|w\|^2 \leq 1} C_{BF}(w, P).
\]

The problem above is equivalent to a max-min-fair (MMF) problem

\[
(\text{MMF}) \quad \max_{\|w\|^2 \leq 1} \min_{i=1,\ldots,M} |h_i^H w|^2.
\]

- (MMF) is NP-hard in general [Sidiropoulos-Davidson-Luo’06].

- Existing design methods (all suboptimal in general):
  - sequential quadratic programming [Sun-Liu’04]
  - semidefinite relaxation (SDR) [Sidiropoulos-Davidson-Luo’06]
  - successive optimization [Hunger-Schmidt-Joham-Schwing-Utschick’07], [Kim-Love-Park’11]
Semidefinite Relaxation for Multicast Beamforming

- By using
  \[ W = w w^H \iff W \succeq 0 \text{ and } \text{rank}(W) \leq 1, \]
  we can rewrite (MMF) as
  \[
  \begin{align*}
  \text{(MMF)} & \quad \max_{W \in \mathbb{H}^N} \min_{i=1,\ldots,M} \text{Tr}(W h_i h_i^H) \\
  & \quad \text{s.t. } \text{Tr}(W) \leq 1, \quad W \succeq 0, \quad \text{rank}(W) \leq 1.
  \end{align*}
  \]

- SDR drops the nonconvex constraint \( \text{rank}(W) \leq 1 \) to obtain a convex relaxation
  \[
  \begin{align*}
  \text{(SDR)} & \quad \max_{W \in \mathbb{H}^N} \min_{i=1,\ldots,M} \text{Tr}(W h_i h_i^H) \\
  & \quad \text{s.t. } \text{Tr}(W) \leq 1, \quad W \succeq 0,
  \end{align*}
  \]
  which is a semidefinite program (SDP).

- Observation: (SDR) is identical to (MC),
  \[ \max_{\Sigma \succeq 0, \text{Tr}(\Sigma) \leq P} \min_{i=1,\ldots,M} \log(1 + \text{SNR}_i). \]
Optimality of SDR

• If an optimal solution $W^*$ to (SDR) satisfies $W^* = \hat{w}\hat{w}^H$ (or rank($W^*) \leq 1$), then $\hat{w}$ is optimal to (MMF).

• By SDP rank reduction [Huang-Zhang’07], we know that

$\text{When } M \leq 3, \text{ a rank-1 solution } W^* \text{ to (SDR) exists. Such a } W^* \text{ can be obtained algorithmically.}$

• Implications:
  – Since (SDR) and (MC) are equivalent, BF is an MC-optimal transmit strategy for three users or less.
  – The optimal BF solution can be obtained by solving (SDR).
Gaussian Randomization

• Consider $M \geq 3$, and instances for which $\text{rank}(W^*) > 1$.

• Let $\text{SNR}_{\text{min}}(W) = \min_{i=1,\ldots,M} \text{Tr}(W_h^i h_i^H)$. A Gaussian randomization procedure can be used to generate a feasible solution to (MMF).

\begin{center}
given an SDR solution $W^*$, and a number of randomizations $L$. 
for $j = 1, \ldots, L$
\hspace{1em} generate $\xi_j \sim \mathcal{CN}(0, W^*)$, and let $\hat{w}_j = \xi_j/\|\xi_j\|$
\end{center}

end
\begin{center}
output $\hat{w} = \hat{w}_{j^*}$, where $j^* = \arg \min_{j=1,\ldots,L} \text{SNR}_{\text{min}}(\hat{w}_j \hat{w}_j^H)$.
\end{center}

• Appealing features of SDR + Gaussian randomization:
  - it is empirically found to provide good approximations, especially for small to moderate number of users;
  - there is a theoretical result that supports this empirical observation.
Gaussian Randomization (Cont’d)

• Recall \( \overline{\text{SNR}}_{\text{min}}(W) = \min_{i=1,\ldots,M} \text{Tr}(Wh_ih_i^H) \).

• It is proven that [Luo-Sidiropoulos-Tseng-Zhang’07]

\[
\begin{align*}
\text{With probability at least } 1 - (5/6)^L, \text{ the Gaussian randomization solution } \hat{w} \text{ satisfies} \\
\overline{\text{SNR}}_{\text{min}}(\hat{w}\hat{w}^H) \geq \frac{1}{8M} \overline{\text{SNR}}_{\text{min}}(W^*).
\end{align*}
\]

• Implication: For \( M > 3 \), SDR-based BF has a worst-case rate gap

\[
C_{\text{MC}}(P) - C_{\text{BF}}(\hat{w}, P) \leq \log \left( \frac{1 + \overline{\text{SNR}}_{\text{min}}(W^*)P}{1 + \overline{\text{SNR}}_{\text{min}}(W^*)P/(8M)} \right) \approx \log(8M),
\]

for large \( P \).
Rank-2 Beamformed
Alamouti Space-Time Block Coding
Motivation

• Recall that the rationale of SDR is to use a rank-unconstrained SDP to find a rank-1 transmit covariance $W$, which can be physically realized by BF.

• A natural question arises: Can we do rank-2 SDR?

• Suppose that we want to do this. Then, there are two aspects to consider:
  – From a realizable physical-layer design viewpoint, what kind of transmit strategies would fit in?
  – From an optimization viewpoint, how to proceed with solution generation, and what is its theoretical performance?

• Answer:
  – a combination of BF and the Alamouti space-time block code;
  – the SDP rank reduction theory in [So-Ye-Zhang’08].
The Alamouti Space-Time Block Code

• Let $s = \begin{bmatrix} s_1, s_2 \end{bmatrix}^T$. The Alamouti code is

$$C(s) = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}.$$ 

• arguably the most famous code in space-time coding.

• Features:
  – orthogonal: $C(s)C^H(s) = \|s\|^2 I$;
  – easy to detect $s$;
  – simple performance characterization;
  – ideal choice for isotropic transmission in $2 \times 1$ MISO channels.

• Extensions: Beamformed Alamouti coding, often for point-to-point MIMO [Jöngren-Skoglund-Ottersten’02], [Pascual-Iserte-Palomar-et al.’06], ...

• Recent development: Beamformed Alamouti for multicasting [Wu-Ma-So’12], [Wen-Law-Alabed-Pesavento’12].
BF Alamouti System Model

- Parse \( x(t) \) into blocks via \( X(n) = [ x(2n) \ x(2n + 1) ] \). In block \( n \), we transmit \( s(n) = [ s(2n) \ s(2n + 1) ]^T \) by

\[
X(n) = \sqrt{P}BC(s(n)),
\]

where \( B \in \mathbb{C}^{N \times 2} \) is a transmit beamforming matrix, and

\[
C(s) = \begin{bmatrix}
s_1 & s_2 \\
-s_2^* & s_1^*
\end{bmatrix}
\]

is the Alamouti space-time code.

- By utilizing the special structure of the Alamouti code, rx signals can be equivalently turned to SISO models with SNRs

\[
\text{SNR}_i = P\bar{h}_i^HBB^Hh_i,
\]

and hence its achievable multicast rate (under ideal channel coding on \( s(t) \)) is

\[
C_{BF-ALAM}(B, P) = \min_{i=1,\ldots,M} \log(1 + P\bar{h}_i^HBB^Hh_i).
\]
Rank-2 Beamforming Problem

• Again, the problem is to maximize the multicast rate

\[(BF-ALAM) \quad \max_{B \in \mathbb{C}^{N \times 2}, \text{Tr}(BB^H) \leq 1} C_{BF-ALAM}(B, P).\]

• By observing that

\[W = BB^H \iff W \succeq 0 \text{ and } \text{rank}(W) \leq 2,\]

(BF-ALAM) can be reformulated as

\[
\max_{W \in \mathbb{H}^N} \min_{i=1,\ldots,M} \text{Tr}(Wh_ih_i^H) \\
\text{s.t. } \text{Tr}(W) \leq 1, \quad W \succeq 0, \quad \text{rank}(W) \leq 2.
\]

• Let us do the same trick—dropping the rank constraint. Then we get a relaxation

\[(SDR) \quad \max_{W \in \mathbb{H}^N} \min_{i=1,\ldots,M} \text{Tr}(Wh_ih_i^H) \\
\text{s.t. } \text{Tr}(W) \leq 1, \quad W \succeq 0,
\]

which is the same SDR as in the previous BF!
Optimality of the SDR-based BF Alamouti Scheme

- Let $\mathbf{W}^*$ be a solution to (SDR).

- If $\mathbf{W}^*$ satisfies $\mathbf{W}^* = \mathbf{\hat{B}\hat{B}}^H$, or $\text{rank}(\mathbf{W}^*) \leq 2$, then $\mathbf{\hat{B}}$ is optimal to (BF-ALAM).

- By SDP rank reduction [Huang-Zhang’07], we deduce that

  When $M \leq 8$, a rank-2 solution $\mathbf{W}^*$ to (SDR) exists. Such a $\mathbf{W}^*$ can be obtained algorithmically.

- **Implication:** The BF Alamouti scheme is MC-optimal for eight users or less. The optimal rank-2 BF solution can be obtained by solving (SDR).

- **Comparison:** For BF, we may guarantee the same optimality only for three users or less.
Gaussian Randomization

- Now consider $M \geq 8$, and instances for which $\text{rank}(W^*) > 2$.

- Can we do Gaussian randomization for rank-2 $W$?

- The answer is yes.

Given an SDR solution $W^*$, and a number of randomizations $L$.

for $j = 1, \ldots, L$

generate $\xi_{1,j}, \xi_{2,j} \sim \mathcal{CN}(0, W^*)$, and define $\tilde{B}_j = [\xi_{1,j} \xi_{2,j}]$.

let $\hat{B}_j = \tilde{B}_j / \sqrt{\text{Tr}(\tilde{B}_j \tilde{B}_j^H)}$.

end

Output $\hat{B} = \hat{B}_{j^*}$, where $j^* = \arg \min_{j=1,\ldots,L} \text{SNR}_{\text{min}}(\hat{B}_j \hat{B}_j^H)$.

- The worst-case approximation accuracy can also be proven.
Gaussian Randomization (Cont’d)

• Recall $\overline{\text{SNR}}_{\min}(W) = \min_{i=1,\ldots,M} \text{Tr}(Wh_ih_i^H)$.

• It is proven that [Wu-Ma-So’12]

> With probability at least $1 - (5/6)^L$, the Gaussian randomization solution $\hat{B}$ satisfies

$$\overline{\text{SNR}}_{\min}(\hat{B}\hat{B}^H) \geq \frac{1}{12.22\sqrt{M}} \overline{\text{SNR}}_{\min}(W^*) .$$

• Implication: For $M > 8$, the SDR-based BF Alamouti scheme has a worst-case rate gap

$$C_{MC}(P) - C_{BF-ALAM}(\hat{w}, P) \leq \log \left( \frac{1 + \overline{\text{SNR}}_{\min}(W^*)P}{1 + \overline{\text{SNR}}_{\min}(W^*)P/(12.22\sqrt{M})} \right) \approx \log(12.22\sqrt{M}), \quad \text{for large } P,$$

which is better than the BF rate gap, $\log(8M)$, for $M > 8$. 
Simulation Results: Multicast Rate Versus Power

8 tx antennas, 32 users, 1000 channel realizations, number of randomizations of SDR = 30MN.
Simulation Results: Multicast Rate Versus No. of Users

8 tx antennas, $P = 3$ dB, 1000 channel realizations, number of randomizations of SDR = $30MN$. 
Simulation Results: Bit Error Rate

8 tx antennas, 32 users, QPSK, BICM, rate-1/3 Turbo code with a code length 2880.
Simulation Results: Bit Error Rate (Cont’d)

8 tx antennas, 16 users, QPSK, BICM, rate-1/3 Turbo code with a code length 2880.
Can We Do Rank-$r$ Beamforming, $r \geq 3$?

- From an optimization viewpoint, yes.
  - For example, we can consider rank-3 SDR, wherein an effective SNR loss of $O(M^{1/3})$ can be proven.

- From a realizable physical layer viewpoint, not so sure (if not impossible).
  - A generalization of the Alamouti code is the class of orthogonal space-time block codes (OSTBCs).
  - Full-rate OSTBCs do not exist for $r > 2$ [Liang-Xia’03].
  - For example, for $r = 3$, the maximal rate is $3/4$, and the code is

$$
C(s) = \begin{bmatrix}
  s_1 & -s_2^* & -s_3^* & 0 \\
  s_2 & s_1^* & 0 & -s_3^* \\
  s_3 & 0 & s_1^* & s_2^*
\end{bmatrix}.
$$

The BF-OSTBC achievable multicast rate is

$$
C_{BF-OSTBC}(B, P) = \min_{i=1,\ldots,M} \frac{3}{4} \log \left( 1 + \frac{4}{3} P h_i^H B B^H h_i \right),
$$

where the constant $3/4$ can outweigh the SNR benefit from rank-3 BF.
Stochastic Beamforming
Motivation

• Let us come back to the question of rank-$r$ beamforming for a general $r$.

• Designing an effective rank-$r$ beamforming scheme, say, via space-time coding or spatial multiplexing, appears to be nontrivial.

• Can we consider virtually rank-$r$ beamforming?
Motivation

- Let us come back to the question of rank-$r$ beamforming for a general $r$.  
- Designing an effective rank-$r$ beamforming scheme, say, via space-time coding or spatial multiplexing, appears to be nontrivial.  
- **Can we consider virtually rank-$r$ beamforming?**  
- Let’s swing.
Swinging the Beamformer

We propose a random-in-time BF strategy, called *stochastic beamforming* (SBF).
**Swinging the Beamformer**

We propose a random-in-time BF strategy, called *stochastic beamforming (SBF)*.
We propose a random-in-time BF strategy, called *stochastic beamforming (SBF)*.

![Diagram of Swinging the Beamformer](image)
System Model for Stochastic Beamforming

• Consider a transmit scheme

\[ x(t) = \sqrt{P}w(t)s(t), \quad t = 1, 2, \ldots, T, \]

where \( w(t) \in \mathbb{C}^N \) is a random-in-time BF vector.

• Intuition: If the generation of \( w(t) \) is such that

\[ \mathbb{E}[w(t)w^H(t)] = W^*, \]

then we may get good performance.


System Model for Stochastic Beamforming

- Consider a transmit scheme

\[ x(t) = \sqrt{P} w(t) s(t), \quad t = 1, 2, \ldots, T, \]

where \( w(t) \in \mathbb{C}^N \) is a random-in-time BF vector.

- **Intuition:** If the generation of \( w(t) \) is such that

\[ \mathbb{E}[w(t)w^H(t)] = W^*, \]

then we may get good performance.

(Ken, you are contradicting yourself. You said \( \mathbb{E}[w(t)w^H(t)] = W^* \iff \) multicast capacity (MC) achieving!)
System Model for Stochastic Beamforming (Cont’d)

- At the rx sides, we have

\[ y_i(t) = \sqrt{P} h_i^H w(t) s(t) + n_i(t), \quad t = 1, 2, \ldots, T. \]

This means that the SNRs fluctuates in time, with

\[ \text{SNR}_i(t) = P |h_i^H w(t)|^2. \]
• Time-varying SNRs $\text{SNR}_i(t) = P|\mathbf{h}_i^H \mathbf{w}(t)|^2$ bring about more problems.
  – The SNRs can be high sometimes, but can also be low (deep fades).
  – Uncoded BERs are dominated by deep fade instances.

• But what about channel coding for “averaging out” the deep fades?
Fact: Consider a fast-fading SISO channel

\[ y(t) = \sqrt{P} \alpha(t) \cdot s(t) + n(t), \]

where \( \alpha(t) \) is random. Assume no CSIT. The channel capacity is

\[ C(P) = \mathbb{E}_\alpha[\log(1 + \alpha P)]. \]

The capacity may practically be approached by applying a near-ideal scalar channel code to \( s(t) \).
SBF Multicast Rate

- Assuming ideal channel coding of $s(t)$, the SBF achievable multicast rate is

$$C_{SBF}(P) = \min_{i=1,\ldots,M} \mathbb{E}_{w \sim D}\left[\log(1 + Ph_i^Hww^Hh_i)\right].$$

where we use the random variable $w$ to denote the randomly generated BF vector $w(t)$, and $D$ is the SBF distribution, which satisfies $\mathbb{E}_{w \sim D}[\|w\|^2] \leq 1$.

- How to design $D$?
  - Optimal design of $D$ may be too difficult.
  - Use what we are good at—engineering intuition.
Gaussian SBF

- Generate the SBF vectors by

\[ w \sim C \mathcal{N}(0, W^*). \]

- easy to generate;
- achieve the MC-optimal tx covariance \( \mathbb{E}[w(t)w^H(t)] = W^*; \)
  (be careful, this does not imply capacity being achieved!)
- disadvantage: \( \|w\| \) has a large spread; poor peak-to-average power ratio.
Elliptic SBF

- Generate the SBF vectors by

\[ w = \frac{L^H \alpha}{\|\alpha\|/\sqrt{r}}, \quad \alpha \sim \mathcal{CN}(0, I_r), \]

where \( r = \text{rank}(W^*) \) and \( L \in \mathbb{C}^{r \times N} \) is a square root decomposition of \( W^* \), i.e., \( L^H L = W^* \).

- also easy to generate;
- follow a complex elliptic distribution;
- achieve the MC-optimal tx covariance \( \mathbb{E}[w(t)w^H(t)] = W^* \);
- \( \|w\|^2 \in [r \lambda_{\text{min}}^+(W^*), r \lambda_{\text{max}}(W^*)] \) with probability 1; less power spread than Gaussian SBF.
Bingham SBF

• Generate the SBF vectors by

\[ w = \frac{L^H \alpha}{\|L^H \alpha\|}, \quad \alpha \sim \mathcal{C} \mathcal{N}(0, I_r). \]

where \( r = \text{rank}(W^*) \) and \( L \in \mathbb{C}^{r \times N} \) satisfies \( L^H L = W^* \).

– follow a complex Bingham distribution;
– may not achieve the MC-optimal tx covariance;
– \( \|w\| = 1 \); zero power spread!
Simulation Results: Multicast Rate Versus Power

8 tx antennas, 32 users, 1000 channel realizations, number of randomizations of SDR = 30MN.
Simulation Results: Multicast Rate Versus No. of Users

8 tx antennas, $P = 3\text{dB}$, 1000 channel realizations, number of randomizations of SDR = $30MN$. 
Simulation Results: Bit Error Rate

8 tx antennas, 32 users, QPSK, BICM, rate-1/3 Turbo code with a code length 2880.
Simulation Results: Bit Error Rate (Cont’d)

![Graph showing Bit Error Rate (BER) results.]

8 tx antennas, 16 users, QPSK, BICM, rate-1/3 Turbo code with a code length 2880.
Performance of Gaussian SBF

• The Gaussian SBF multicast rate is equivalent to an ergodic SISO capacity under Rayleigh fading [Alouini-Goldsmith’99].

• Explicit expression of the Gaussian SBF multicast rate:

\[ C_{\text{SBF}}^{\text{Gauss}}(P) = e^{1/(\rho_{\text{min}}P)}E_1(1/(\rho_{\text{min}}P)), \]

where \( E_1(x) = \int_1^{\infty} t^{-1}e^{-xt}dt, \ x \geq 0, \) is the exp. integral of the 1st order.

• More importantly,

\[ C_{\text{MC}}(P) - C_{\text{SBF}}^{\text{Gauss}}(P) \leq \gamma = 0.5772 \quad \text{for all } P \geq 0. \]

• Implication: The Gaussian SBF rate loss is at most 0.8314 bits/s/Hz (\( \gamma / \log(2) = 0.8314 \)), \textit{irrespective of the number of users}.

• Recall the rate losses of BF and BF Alamouti increase with the number of users.
Performance of Elliptic SBF

- Explicit expression of the elliptic SBF multicast rate:

\[
C_{SBF}^{Ellip}(P) = \left(1 + \frac{1}{r \rho_{min} P}\right)^{r-1} \left[ \log(1 + r \rho_{min} P) - \sum_{k=1}^{r-1} \frac{1}{k} \right. \\
\left. \sum_{k=1}^{r-1} \binom{r-1}{k} \frac{(-1)^k}{k(1 + r \rho_{min} P)^k} \right].
\]

- Moreover,

The elliptic SBF multicast rate gap satisfies

\[
C_{MC}(P) - C_{SBF}^{Ellip}(P) \leq \sum_{k=1}^{r-1} \frac{1}{k} - \log(r) \quad \text{for all } P \geq 0.
\]

- For \( r = 2 \), the gap is 0.4428 bits/s/Hz; for \( r = 4 \), 0.6449 bits/s/Hz; for \( r \to \infty \), 0.8314 bits/s/Hz (the same as Gaussian SBF).
Performance of Bingham SBF

- The explicit expression of the Bingham SBF multicast rate can be derived, but is too complicated to provide insight.

- By stochastic majorization, we prove that

\[
C_{MC}(P) - C_{SBF}^{Bing}(P) \leq \sum_{k=1}^{r-1} \frac{1}{k} - \log(r) \quad \text{for all } P \geq 0.
\]

- (Surprisingly) While Bingham SBF does not satisfy MC-optimal tx covariance, it has the same worst-case rate gap as elliptic SBF!
Combining SBF and Alamouti Space-Time Coding

- A more powerful way to use SBF is to perform it in a rank-2 manner.
- We combine SBF and the beamformed Alamouti scheme, and show that

\[
C_{MC}(P) - C_{SBF-ALAM}^{Gauss}(P) \leq \log(2) + \gamma - 1 = 0.2703,
\]
\[
C_{MC}(P) - C_{SBF-ALAM}^{Ellip}(P) \leq \sum_{k=1}^{2r-1} \frac{1}{k} - \log(r) - 1,
\]
\[
C_{MC}(P) - C_{SBF-ALAM}^{Bing}(P) \leq \sum_{k=1}^{2r-1} \frac{1}{k} - \log(r) - 1
\]

for all \( P \geq 0 \).

- The worst-case rate gap this time is 0.39 bits/s/Hz \((0.2703/\log(2) = 0.39)\); i.e., approach the capacity to within 1/2 bits!
Simulation Results: Multicast Rate Versus Power

8 tx antennas, 32 users, 1000 channel realizations, number of randomizations of SDR = 30MN.
Simulation Results: Multicast Rate Versus No. of Users

8 tx antennas, $P = 3$dB, 1000 channel realizations, number of randomizations of SDR = $30MN$. 
Simulation Results: Bit Error Rate

8 tx antennas, 32 users, QPSK, BICM, rate-1/3 Turbo code with a code length 2880.
Simulation Results: Bit Error Rate (Cont’d)

- SISO bound
- BF (via SDR)
- Gaussian SBF
- Elliptic SBF
- Bingham SBF
- BF Alamouti (via SDR)
- Gaussian SBF Alamouti
- Elliptic SBF Alamouti
- Bingham SBF Alamouti

8 tx antennas, 16 users, QPSK, BICM, rate-1/3 Turbo code with a code length 2880.
Conclusion

- Two realizable approaches for physical-layer multicasting have been proposed.

- By using a beamformed Alamouti scheme, we can achieve a provable better rate or SNR performance than the previous beamforming scheme.

- By using random-in-time beamforming schemes, we can achieve a constant rate loss to within 1 or 1/2 bits.
Conclusion

• Two realizable approaches for physical-layer multicasting have been proposed.

• By using a beamformed Alamouti scheme, we can achieve a provably better rate or SNR performance than the previous beamforming scheme.

• By using random-in-time beamforming schemes, we can achieve a constant rate loss to within 1 or 1/2 bits.

Thank you!
References


